

Naval Surface Warfare Center

Carderock Division

West Bethesda, MD 20817-5700

NSWCCD-70-TR—2003/115 November 2003

Signatures Directorate

Technical Report

Are the Energy Analysis (EA) and the Statistical Energy Analysis (SEA) Compatible?

by

G. Maidanik

K. J. Becker

[Work supported by ONR and In-House Fundings.]

20031217 068



Approved for public release; distribution is unlimited.

REPORT DOCUMENTATION PAGE				Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.					
1. REPORT DATE (DD-MM-YYYY) 7-Nov-2003		2. REPORT TYPE Final		3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE Are the Energy Analysis (EA) and the Statistical Energy Analysis (SEA) Compatible?				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) G. Maidanik, K. J. Becker				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) AND ADDRESS(ES) Naval Surface Warfare Center Carderock Division 9500 Macarthur Boulevard West Bethesda, MD 20817-5700				8. PERFORMING ORGANIZATION REPORT NUMBER NSWCCD-70-TR-2003/115 NOV 2003	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) Attn ONR 334 Chief of Naval Research Ballston Centre Tower One 800 North Quincy Street Arlington, VA 22217-5660				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT Originally the statistical energy analysis (SEA) was restricted to a low coupling loss factor, at least, lower than the loss factor of the (adjunct) dynamic system to which the externally driven (master) dynamic system was coupled. The coupling loss factor of reference is that from the adjunct dynamic system to the master dynamic system. With the advent of structural fuzzies, as introduced by Soize and subsequently interpreted by a number of researchers, questions relating not only to the validity of the conservation of energy arose, but also arose were questions relating to the coupling loss factors, to the loss factors and to the external input powers. In trying to decipher, in terms of (SEA), some of these questions, a number of surprising answers emerged which casts doubts on the universal validity of (SEA). In small part this report attempts to warn the noise control engineers that as valuable as (SEA) is, it has fundamental limitations and that these limitations are not merely and strictly a question of frequency regions; i.e., high-, mid- and low-frequencies.					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UL	18. NUMBER OF PAGES 28	19a. NAME OF RESPONSIBLE PERSON G. Maidanik
a. REPORT UNCLASSIFIED	b. ABSTRACT UNCLASSIFIED	c. THIS PAGE UNCLASSIFIED			19b. TELEPHONE NUMBER (include area code) 301-227-1282

Contents

Page

Report Document Page	i
Contents	ii
Abstract	1
Introduction	2
Viewgraphs	8
References	15
Appendix A	A-1
Appendix B	B-1

ABSTRACT

Originally the statistical energy analysis (SEA) was restricted to low coupling loss factors, at least, lower than the corresponding loss factors of the (adjunct) dynamic system to which the externally driven (master) dynamic system was coupled. The coupling loss factors of reference are those from the adjunct dynamic system to the master dynamic system. Something happened on the way and this restriction was lost and never examined again, at least, until now. With the advent of structural fuzzies, as introduced by Soize and subsequently interpreted by a number of researchers, questions relating not only to the validity of the conservation of energy arose, but also arose were questions relating to the coupling loss factors, to the loss factors and to the external input powers. In trying to decipher, in terms of (SEA), some of these questions, a number of surprising answers emerged which casts doubts on the universal validity of (SEA). In this vein, for example, the authors of this report have written a few papers defining and redefining various loss factors. These authors found that the definitions of loss factors are as elusive as are the definitions of radiation efficiencies. Both, loss factors and radiation efficiencies, are parameters that require redefinitions in order to convince the noise control engineers that they are using them correctly when engaged in a particular noise control task. To do otherwise invites misrepresentations and false claims. In small part this report attempts to warn the noise control engineers that as valuable as (SEA) is, it has fundamental limitations and that these limitations are not merely and strictly a question of frequency regions; i.e., high-, mid- and low-frequencies, even though it is recognized that some of the limitations, herein considered, may be relieved by introducing these frequency divisions on the validity of (SEA).

INTRODUCTION

This report is a paper intended for oral delivery at the 26th Meeting of the Acoustical Society of America in Austin, Texas and constitutes a part of a key-note address to be given in TBL Noise Class at the Boeing Company in Seattle, Washington.

Sketched in the first viewgraph (V1) is an externally force-driven isolated dynamic system - the *master* dynamic system. The accounting of the response of this dynamic system is conducted in terms of an energy analysis (EA). In this analysis the external force-drive is stated in terms of the external input power $\Pi_e^o(\omega)$, which is a function of the frequency (ω) ; (ω) is the center frequency of a band of width $(\Delta\omega)$. The response is stated in terms of the energy $E_o^o(\omega)$ stored in the master dynamic system. The master dynamic system is defined in terms of the loss factor $\eta_o(\omega)$, the modal density $\nu_o(\omega)$ and the mass (M_o) [1-3]. The loss factor $\eta_o(\omega)$ relates the power dissipated in the master dynamic system to the energy $E_o^o(\omega)$ stored in that dynamic system; namely

$$\Pi_o^o(\omega) = \eta_o(\omega)[\omega E_o^o(\omega)] \quad .(1a)$$

The conservation of energy (power) demands that the dissipated power be equal to the external input power $\Pi_e^o(\omega)$; namely

$$\Pi_e^o(\omega) = \Pi_o^o(\omega) \quad .(1b)$$

The modal density $\nu_o(\omega)$ is the number of modes per unit frequency in the master dynamic system [3]. It follows that the number of modes $N_o(\omega)$ that resides within the frequency bandwidth $(\Delta\omega)$ in this dynamic system is given by

$$N_o(\omega) = \nu_o(\omega) \Delta\omega \quad . (2)$$

From Equations (1) and (2) the modal external input power $\pi_e^o(\omega)$, the modal power $\pi_o^o(\omega)$ dissipated and the modal energy $\varepsilon_o^o(\omega)$ stored may be cast in the forms

$$\Pi_e^o(\omega) = N_o(\omega) \pi_e^o(\omega) \quad ; \quad \pi_o^o(\omega) = N_o(\omega) \pi_o^o(\omega) \quad ; \quad E_o^o(\omega) = N_o(\omega) \varepsilon_o^o(\omega) \quad , (3)$$

respectively.

The master dynamic system is now coupled to an adjunct dynamic system; the coupling may be either mass control (m_c) , stiffness control (k_c) , gyroscopically control (G) or any combination thereof [3,4]. A question arises: What is the influence of this coupling either on the response of the master dynamic system, on the response of the adjunct dynamic system or on the response of the dynamic system as a whole (master + adjunct) [5]? One of the major influences is that a number of loss factors may be appropriately defined [5-8]. As Equation (1) initiated, an appropriate loss factor is one that relates a definitive stored energy to a definitive dissipated power. In turn, the dissipated power must be balanced in a conservation of energy (power) equation.

Sketched in the second viewgraph (V2) are a number of appropriately defined loss factors among them the induced loss factor (η_I). These loss factors, as already stated, relate stored energies to corresponding powers dissipated [1-8].

In the third viewgraph (V3) the conservation of energy (power) is imposed and some of the relationships among various loss factors are stated. Central to some of these relationships is the definition of the global coupling strength $\mathfrak{I}_o^s(\omega)$ [1-3]. The global coupling strength is the ratio of the stored energy $E_s(\omega)$ in the adjunct dynamic system to the corresponding stored energy $E_o(\omega)$ in the master dynamic system

$$\mathfrak{I}_o^s(\omega) = [E_s(\omega) / E_o(\omega)] \quad . (4)$$

Significantly, it is found that the ratio of the induced loss factor $\eta_I(\omega)$, in the master dynamic system, to the indigenous loss factor $\eta_s(\omega)$, in the adjunct dynamic system, is equal to the global coupling strength $\mathfrak{I}_o^s(\omega)$; i.e.,

$$\mathfrak{I}_o^s(\omega) = [\eta_I(\omega) / \eta_s(\omega)] \quad , (5)$$

where $\mathfrak{I}_o^s(\omega)$ is defined in Equation (4).

Sketch in the fourth viewgraph (V4) is the transference from the global to the modal coupling strength; the modal coupling strength is related to the global coupling strength by merely the ratio of the modal density $\nu_o(\omega)$ of the master dynamic system to the modal density $\nu_s(\omega)$ of

the adjunct dynamic system, respectively [3]. Explicitly this relationship is

$$\zeta_o^s(\omega) = [\nu_o(\omega)/\nu_s(\omega)] \mathfrak{I}_o^s(\omega) \quad .(6a)$$

An induced modal overlap parameter $b_I(\omega)$ for the master dynamic system and an indigenous modal overlap parameter $b_s(\omega)$ for the adjunct dynamic system are defined

$$b_I(\omega) = \nu_o[\omega \eta_I(\omega)] \quad ; \quad b_s(\omega) = \nu_o[\omega \eta_s(\omega)] \quad .(7)$$

Then from Equations (5), (6a) and (7) the modal coupling strength $\zeta_o^s(\omega)$ may be cast in the form

$$\zeta_o^s(\omega) = [b_I(\omega)/b_s(\omega)] \quad .(6b)$$

(The modal overlap parameter $\{\nu(\omega)[\omega \eta(\omega)]\}$ simply states the ratio between the frequency width $[\omega \eta(\omega)]$ of a typical mode to the corresponding typical frequency distance $[\nu(\omega)]^{-1}$ between neighboring modes [3].)

In the fifth viewgraph (V5) it is pointed out that a "smoothed out" induced loss factor $\langle \eta_I(\omega) \rangle$ is independent of $\eta_s(\omega)$; notwithstanding that $\eta_I(\omega)$ exhibits modal undulations, that pertain to modes in the adjunct dynamic system, for values of $b_s(\omega)$ that are less than unity. [3,5-15] It

is thus concluded that the "smoothed out" value of the modal coupling strength $\langle \zeta_o^s(\omega) \rangle$ exceeds unity if

$$\langle b_l(\omega) \rangle > b_s, \quad (7a)$$

and is less than unity if

$$\langle b_l(\omega) \rangle < b_s. \quad (7b)$$

(It is to be understood that what is called here the smoothed out value of a quantity is commensurate with Skudrzyk's mean-value for this quantity [10].)

Sketched in the sixth viewgraph (V6) is the derivation of the modal coupling strength $\zeta_o^{sea}(\omega)$ in terms of the statistical energy analysis (SEA) [1-3]. It is argued that $\zeta_o^{sea}(\omega)$, by definition, remains less than unity. Indeed

$$\zeta_o^{sea}(\omega) = \eta_{os}(\omega) [\eta_{os}(\omega) + \eta_s(\omega)]^{-1} < 1 \quad (8)$$

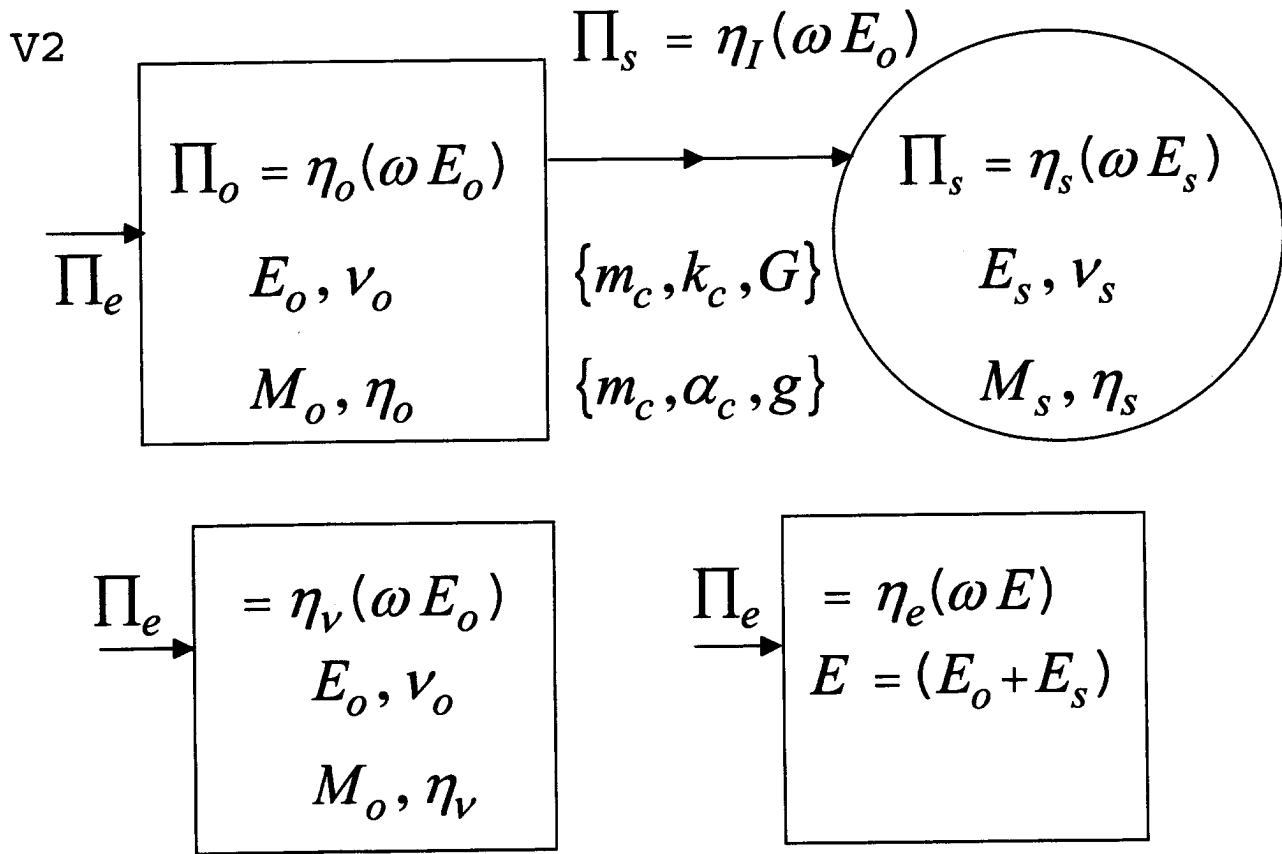
The seventh viewgraph (V7) again emphasizes that the modal coupling strength $\zeta_o^{sea}(\omega)$ in SEA, by definition, is less than unity; $\zeta_o^{sea}(\omega) < 1$. On the other hand, the modal coupling strength $\zeta_o^s(\omega)$ in EA is not so restricted. Then, in order for $\zeta_o^s(\omega)$ to be compatible with $\zeta_o^{sea}(\omega)$, the modal overlap parameter $b_s(\omega)$ of the adjunct dynamic system must exceed

the smoothed-out induced modal overlap parameter $\langle b_I \rangle$ of the master dynamic system; $b_s(\omega) > \langle b_I \rangle$. [cf. Appendix B.] To validate (SEA), $\langle b_I \rangle$ serves as a lower threshold for (b_s) .

$$\begin{array}{c} \Pi_e^o \longrightarrow \boxed{\begin{array}{l} = \Pi_o^o = \eta_o(\omega E_o^o) \\ E_o^o, \nu_o \\ M_o, \eta_o \end{array}} \end{array}$$

(η_o) the loss factor of the master
dynamic system

The master dynamic system is coupled to an
adjunct dynamic system resulting in the
definition of a number of loss factors,
among them the induced loss factor (η_I)



η_s the loss factor of adjunct dynamic system

η_v the virtual loss factor of the coupled master dynamic system

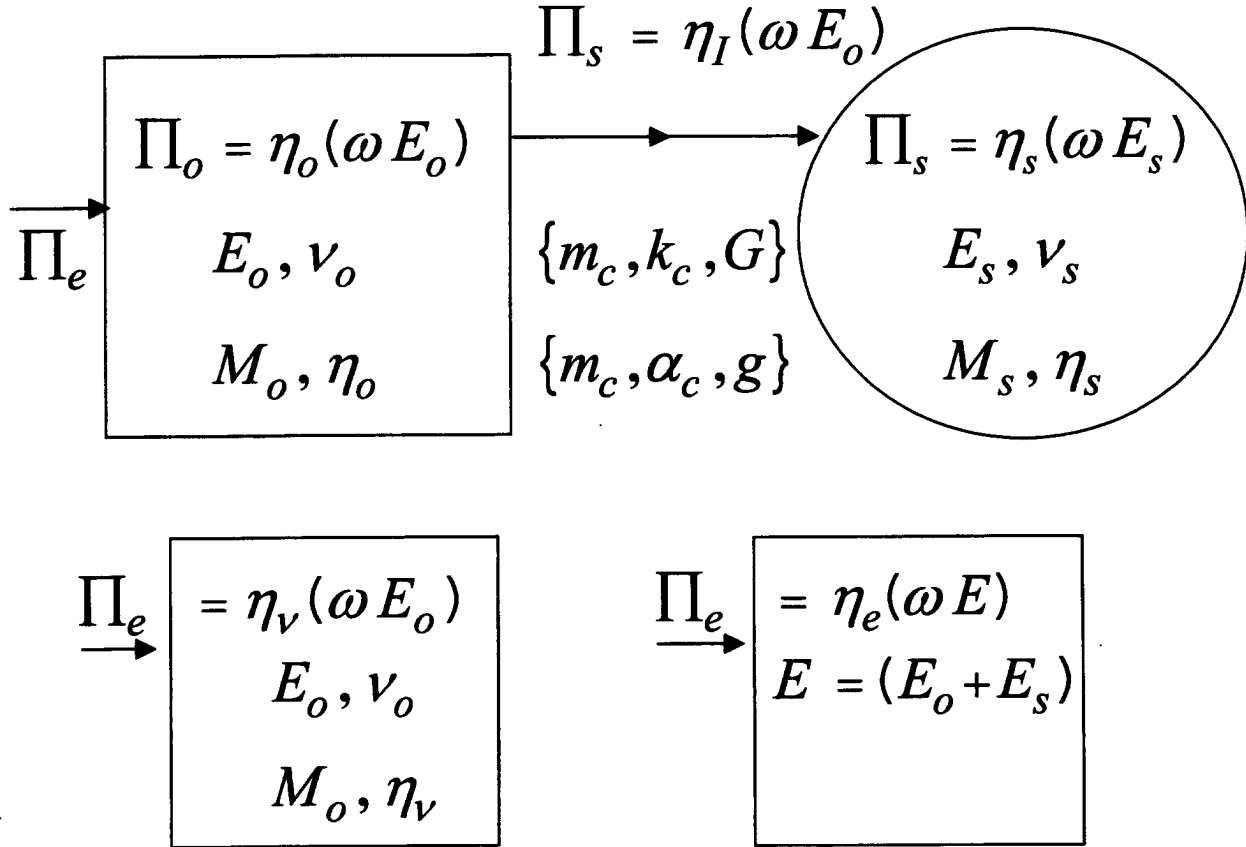
η_e the effective loss factor of the coupled dynamic system (master + adjunct)

$$\Pi_s = \eta_I(\omega E_o) = \eta_s(\omega E_s)$$

η_I the induced loss factor of the master dynamic system; induced by the coupling.

The Conservation of Energy (Power) $\Pi_e = \Pi_o + \Pi_s$

and the relationships among some loss factors

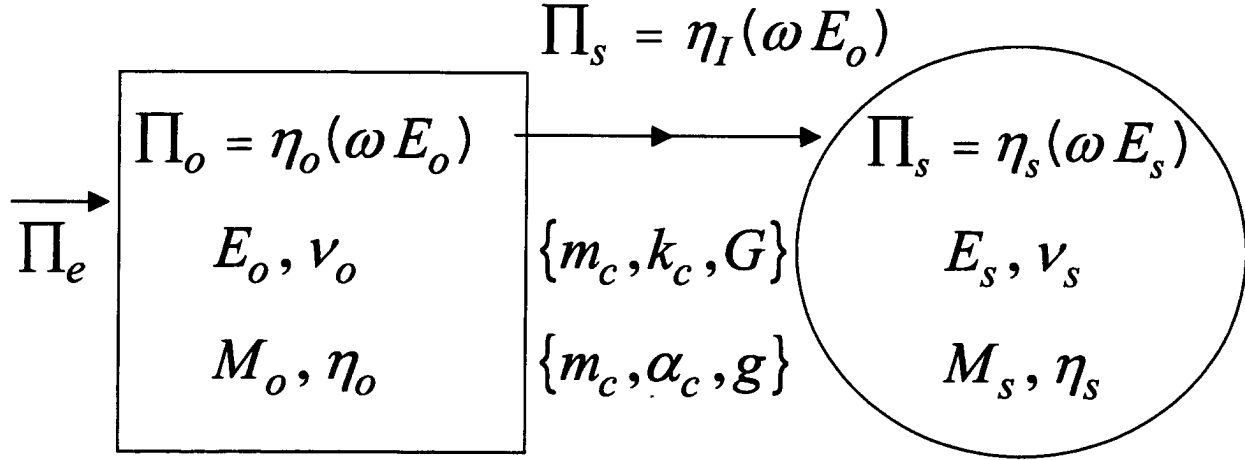


$$\eta_v = \eta_o + \eta_I \quad ; \quad \eta_v = \eta_e(1 + \mathfrak{I}_o^s) \quad ; \quad \mathfrak{I}_o^s = (E_s/E_o)$$

\mathfrak{I}_o^s the global coupling strength

$$\eta_I(\omega E_o) = \eta_s(\omega E_s) \quad ; \quad (\eta_I / \eta_s) = \mathfrak{I}_o^s$$

The modal coupling strength and the modal overlap parameter



$$\Pi_s = \eta_I(\omega E_o) = \eta_s(\omega E_s) ; \quad \mathfrak{I}_o^s = (E_s / E_o) ; \quad (\eta_I / \eta_s) = \mathfrak{I}_o^s$$

\mathfrak{I}_o^s the global coupling strength

$$(\nu_o / \nu_s) \mathfrak{I}_o^s = \zeta_o^s \quad ; \quad (\nu_o / \nu_s) (\eta_I / \eta_s) = (b_I / b_s) = \zeta_o^s$$

ζ_o^s the modal coupling strength

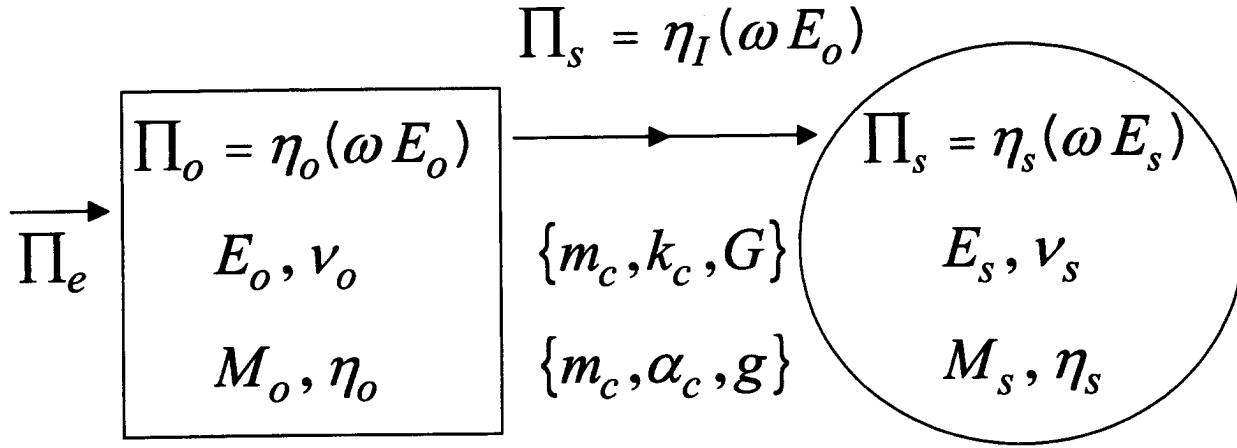
$(b_I) = [\nu_o(\omega \eta_I)]$ the induced modal overlap

parameter of the master dynamic system

$(b_s) = [\nu_s(\omega \eta_s)]$ the modal overlap parameter of

the adjunct dynamic system

The "smoothed out" induced loss factor $\langle \eta_I \rangle$



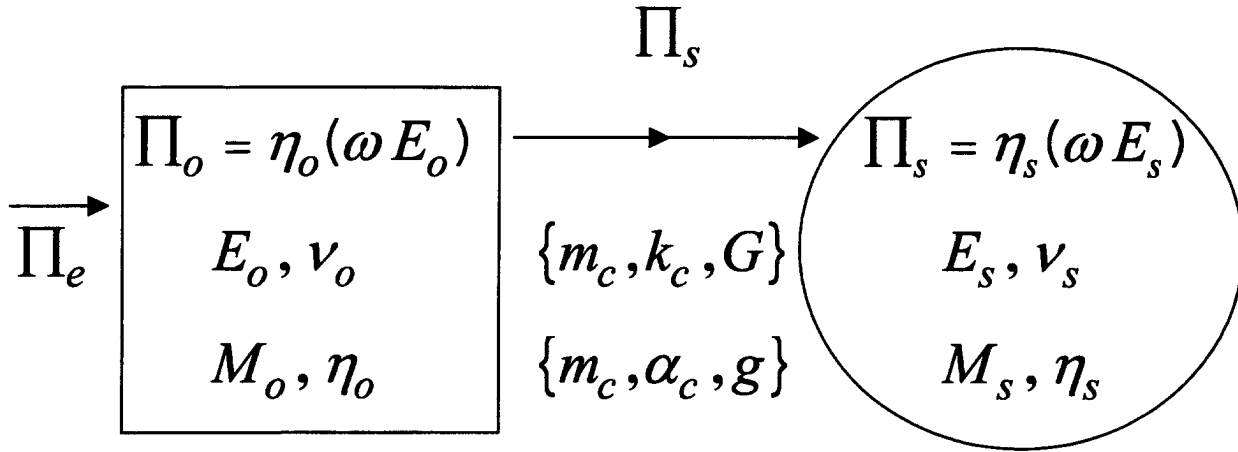
It is found that $\langle \eta_I \rangle$ is independent of (η_s) although the modal undulations in (η_I) are dependent on (η_s) through the value of (b_s) . There are no modal undulations in the adjunct dynamic system if $b_s \geq 1$.

Thus $[v_o(\omega \langle \eta_I \rangle)] [v_s(\omega \eta_s)]^{-1} = (\langle b_I \rangle / b_s) = \langle \zeta_o^s \rangle$

There is no restriction on the value of $\langle \zeta_o^s \rangle$; noting that $\langle \eta_I \rangle$ is the larger, the stronger the coupling. If the coupling is strong and the loss factor (η_s) of the adjunct dynamic system is small, $\langle \zeta_o^s \rangle$ may exceed unity.

V6

Under SEA



$$\eta_s(\omega E_s) = \Pi_s = [\eta_{so}(\omega E_o) - \eta_{os}(\omega E_s)]$$

Hence

$$(\eta_s + \eta_{os})(\omega E_s) = \eta_{so}(\omega E_o) \quad ; \quad \mathfrak{J}_o^{sea} = (E_s / E_o)$$

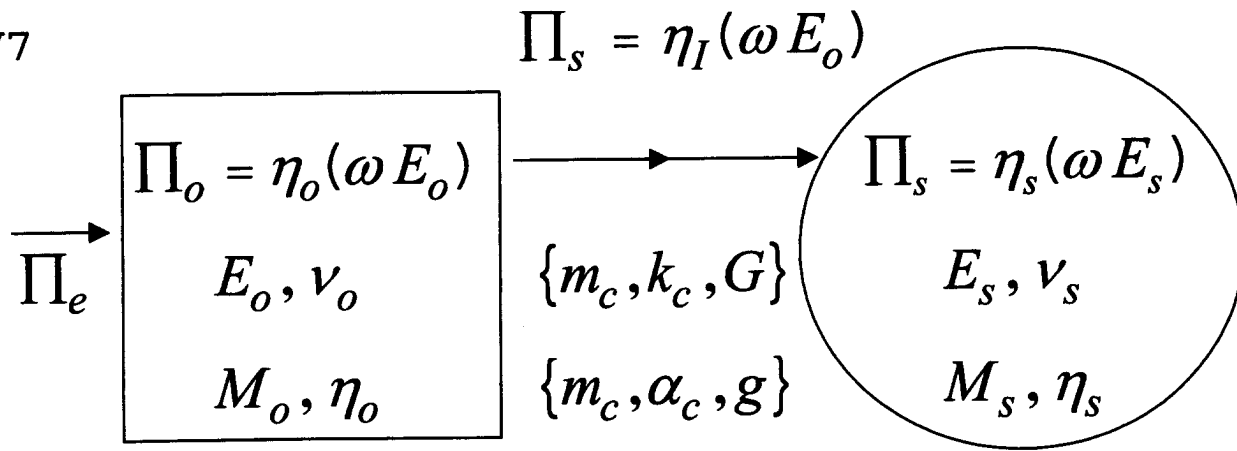
$$\mathfrak{J}_o^{sea} = \eta_{so}(\eta_s + \eta_{os}) \quad ; \quad (\eta_{so} / \eta_{os}) = (\nu_s / \nu_o)$$

$$\zeta_o^{sea} = \eta_{os}(\eta_{os} + \eta_s)^{-1} < 1 \quad ; \quad \mathfrak{J}_o^{sea} = (\nu_s / \nu_o) \zeta_o^{sea}$$

Again

$$\zeta_o^{sea} < 1$$

V7



A tenet of SEA

$$\eta_s(\omega E_s) = \Pi_s = [\eta_{so}(\omega E_o) - \eta_{os}(\omega E_s)]$$

$$\zeta_o^{sea} = \eta_{os}(\eta_{os} + \eta_s)^{-1} < 1$$

A tenet of EA

$$\eta_s(\omega E_s) = \Pi_s = \eta_I(\omega E_o) \quad ; \quad \langle \zeta_o^s \rangle = (\langle b_I \rangle / b_s)$$

Thus for ζ_o^{sea} and $\langle \zeta_o^s \rangle$ to be compatible (b_s) must exceed $\langle b_I \rangle$. Then $\langle b_I \rangle$ constitutes a threshold for (b_s) to validate SEA; $\langle b_I \rangle < b_s$.

References

1. L. Cremer, M. Heckl, and E. Ungar, Structure-Borne Sound, Structural Vibrations and Sound Radiation at Audio Frequencies, 1988, Springer-Verlag, 2nd Ed. Berlin.
2. F. Fahy, Sound and Structural Vibration (Radiation, Transmission and Response, 1985, Academic Press, London.
3. R. H. Lyon, Statistical Energy Analysis of Dynamic Systems: Theory and Applications, 1975, MIT, Cambridge; R. H. Lyon and R. G. Dejung, Theory and Application of Statistical Energy Analysis, 1995, Butterworth-Heinemann, Boston.
4. R. H. Lyon and G. Maidanik 1962, Journal of the Acoustical Society of America, **34**, 623-639. Power flow between linearly coupled oscillators.
5. C. Soize 1986 and 1993, Rech. Aerosp. **1986-3**, 23-48. Probabilistic structural modeling in linear dynamic analysis of complex mechanical systems. Journal of the Acoustical Society of America, **94**, 849-865. A model and numerical method in the medium frequency range for

vibroacoustic predictions using the theory of structural fuzzy.

6. A. Pierce, V. W. Sparrow and D. A. Russell 1995, Journal of Acoustics and Vibration, **117**, 339-348. Fundamental structural-acoustic idealizations for structures with fuzzy internals.
7. M. Strasberg and D. Feit 1996, Journal of the Acoustical Society of America, **99**, 335-344. Vibration of large structures by attached small resonant structures.
8. G. Maidanik and K. J. Becker 1998, Journal of the Acoustical Society of America, **103**, 3184-3195. Various loss factors of a master harmonic oscillator that is coupled to a number of satellite harmonic oscillators.
9. G. Maidanik 2000, Journal of Sound and Vibration, **240**, 717-731. Induced damping by a nearly continuous distribution of nearly undamped oscillators: Linear Analysis.
10. E. Skudrzyk 1980, Journal of the Acoustical Society of America, **67**, 1105-1135. The mean-value method of predicting the dynamic response of complex vibrations.

11. M. J. Brennan 1977, Noise Control Engineering Journal, **45**, 201-207. Wideband vibration neutralizer.
12. R. J. Nagem, I. Veljkovic and G. Sandri 1977, Journal of Sound and Vibration, **207**, 429-434. Vibration damping by a continuous distribution of undamped oscillators.
13. Yu. A. Kobelev 1987, Soviet Physics Acoustics, **33**, 295-296. Absorption of sound waves in a thin layer.
14. G. Maidanik and K. J. Becker 2003, Journal of Sound and Vibration, **266**, 15-32. Dependence of the induced damping on the coupling forms and coupling strengths: Linear Analysis; and Journal of Sound and Vibration, **266**, 33-48. Dependence of the induced damping on the coupling forms and coupling strengths: Energy Analysis.
15. G. Maidanik 2000, Journal of Sound and Vibration, **240**, 717-731. Induced damping by a nearly continuous distribution of nearly undamped oscillators: Linear Analysis.

16. M. Strasberg 1996, Journal of the Acoustical Society of America, **100**, 3456-3459. Continuous structures as 'fuzzy' substructures.
17. G. Maidanik and K. J. Becker 2004, Accepted for publication in the Journal of Sound and Vibration. Induced noise control.

Appendix A

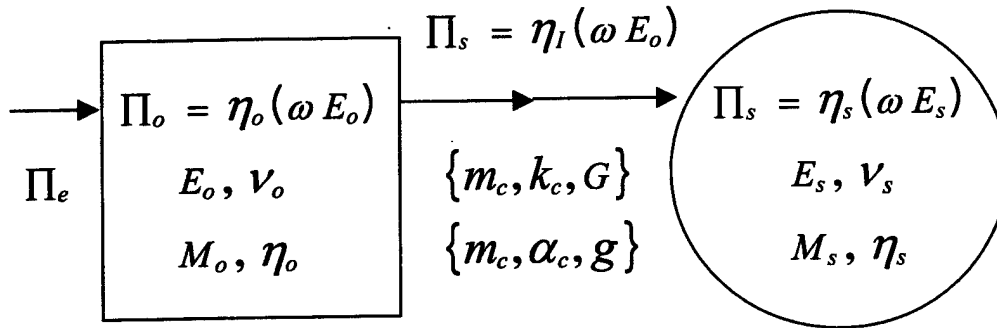
The modal coupling strength in SEA is shown to be the relationship

$$\zeta_o^{sea} = \eta_{os}(\eta_{os} + \eta_s)^{-1} < 1 \quad . (A1)$$

Using the relationship in SEA

$$\langle \eta_I \rangle (\omega E_o) = \eta_s (\omega E_s) = \Pi_s = [\eta_{so} (\omega E_o) - \eta_{os} (\omega E_s)] \quad , (A2)$$

taken off the figure below



one finds the equivalent to Equation (A1) in the form

$$(\langle \eta_I \rangle / \eta_{so}) = \eta_s(\eta_{os} + \eta_s)^{-1} < 1 \quad . (A2)$$

It is noted that both $\langle \eta_I \rangle$ and (η_{so}) are dependent on the strength of the coupling between the master dynamic system and the adjunct dynamic system. Moreover, from Equation (A2), were the coupling loss factor (η_{os}) , from the adjunct dynamic system to the master dynamic system, small compared with the

loss factor (η_s), of the adjunct dynamic system; $\eta_{os} \ll \eta_s$, the smoothed-out induced loss factor is largely equal to the coupling loss factor (η_{so}) [16]. The coupling loss factor (η_{so}) accounts for the transfer of power from the master dynamic system to the adjunct dynamic system.

Appendix B

One may state the relationship between the virtual loss factor (η_v) and the effective loss factor (η_e) in the form

$$\eta_v = \eta_e(1 + \mathfrak{I}_o^s) \quad ; \quad \eta_v = (\eta_I + \eta_o) \quad ; \quad \mathfrak{I}_o^s = (\eta_I / \eta_s) \quad . (B1)$$

From Equation (B1) one may derive

$$(\eta_e - \eta_o)(\eta_s - \eta_e)^{-1} = (\eta_I / \eta_s) = \mathfrak{I}_o^s \quad . (B2)$$

Since (\mathfrak{I}_o^s) is positive definite (including zero) it follows that:

$$\text{If } \eta_s > \eta_o \quad ; \quad \eta_s > \eta_e > \eta_o \quad , (B3a)$$

and

$$\text{if } \eta_s < \eta_o \quad ; \quad \eta_s < \eta_e < \eta_o \quad . (B3b)$$

On the other hand, if the adjunct dynamic system is a *sink*; defined such that $\mathfrak{I}_o^s \equiv 0$, then

$$\eta_e \Rightarrow \eta_v = \eta_I + \eta_o \quad ; \quad \eta_e > \eta_o \quad . (B4)$$

From Equations (B3a) and (B4) one finds that

$$\eta_s > \eta_o + \eta_I > \eta_o \quad . (B5)$$

In this case (η_I) is the additional loss factor that is acquired by the master dynamic system due to its coupling to the sink. Equation (B5) merely states that an adjunct dynamic system that qualifies as a sink would possess a loss factor that exceeds that of the master dynamic system when *coupled* to that sink [17].

INITIAL DISTRIBUTION

Copies		Copies	Code	Name
3	NAVSEA 05T2	1	7020	Strasberg
	1 Taddeo			
	1 Biancardi	1	7030	Maidanik
	1 Shaw			
		1	7204	Niemiec
3	ONR/ONT			
	1 334 Schreppler	1	7205	Dlubac
	1 334 Couchman			
	1 Library	1	7200	Shang
2	DTIC	1	7207	Becker
2	Johns Hopkins University	2	7250	Maga
	1 Green			Diperna
	1 Dickey			
		3	3421	TIC-Carderock
4	ARL/Penn State University			
	1 Koopman			
	1 Hwang			
	1 Hambric			
	1 Conlin			
1	R. H. Lyon, Corp.			
	1 Lyon			
1	MIT			
	1 Dyer			
1	Florida Atlantic University			
	1 Cuschieri			
2	Boston University			
	1 Pierce			
	1 Barbone			

CENTER DISTRIBUTION

1	0112 Barkyoumb
1	7000 Jebsen